## Доказать, что $E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$

$$A = \int_1^2 \vec{F} d\vec{S} = \int_{t_1}^{t_2} (\vec{F}\vec{v}) dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\vec{v} dt = \int_{p_1}^{p_2} \vec{v} d\vec{p} = (\vec{p}\cdot\vec{v}) - \int_{v_1}^{v_2} (\vec{p} d\vec{v}) =$$

$$= (\vec{p}\cdot\vec{v}) - \int_{v_1}^{v_2} \frac{m\vec{v}d\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = (\vec{p}\cdot\vec{v}) - \frac{1}{2}\int_{v_1}^{v_2} \frac{m d\vec{v^2}}{\sqrt{1-\frac{v^2}{c^2}}} = (\zeta = 1 - \frac{v^2}{c^2}) =$$

$$(\vec{p}\cdot\vec{v}) + \frac{mc^2}{2}\int_{\zeta_1}^{\zeta_2} \zeta^{-\frac{1}{2}} d\zeta = (\vec{p}\cdot\vec{v} + mc^2\zeta)|_{\zeta_1}^{\zeta_2} = \left(\frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} + mc^2\sqrt{1-\frac{v^2}{c^2}}\right)\Bigg|_{v_1}^{v_2} =$$

$$\left[\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right]\Bigg|_{v_1}^{v_2} = E_{k_2} - E_{k_1} \implies E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + K_0$$

## Найдём $K_0$

При $v \to 0$:

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + K_0 = \frac{mv^2}{2} = \star$$

$$\frac{d}{dv}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{1}{2c^2}\frac{(-1)2v^1}{\sqrt{1-\frac{v^2}{c^2}}^3}$$

$$\frac{d^2}{dv^2}\frac{1}{\sqrt{1-\frac{v^2}{2c^2}}} = \frac{1}{2c^2}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}^3} + \frac{1}{2c^4}\frac{3v^2}{\sqrt{1-\frac{v^2}{c^2}}^5},$$

Как видно, при $v \to 0$, $E^{(n)}(v) \to C_n$, где $C_n$ - конечное число, которое при умножении на $v^n$ даёт $0$.

$$E(v) = \frac{E(v=0)v^0}{0!} + \frac{E'(v=0)v^1}{1!} + \frac{E''(v=0)v^2}{2!} + \dots$$

$$\star = mc^2\left(1 + \frac{v^2}{2c^2} + \frac{v^2}{4c^2} + \frac{3v^4}{4c^4} + \dots\right) + K_0 = \frac{mv^2}{2}$$

$$mc^2 + \frac{mv^2}{2} + \frac{mv^2}{4} + \frac{3mv^4}{4c^2} + \dots + K_0 = \frac{mv^2}{2}$$

$$mc^2 + 0 + 0 + \dots + K_0 = 0 \implies K_0 = -mc^2,$$

$$E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$$